# More on Transmission Lines and SWR – Part III

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It's a common misconception that SWR is the ratio of the antenna feed-point impedance to the transmission line characteristic impedance. That is only true if the impedance of the antenna is purely resistive, which in the real world is almost never the case. If there is any reactance at all in the antenna feed-point impedance, then the SWR isn't simply the ratio of the impedances.

## SWR and Related Relationships

SWR, forward (incident) power, reflected power, reflection coefficient and return loss were all addressed in Part II of this series. The relationship of these parameters to one another is shown in Table 1.

Table 1. The Relationship Between Power Ratio, Reflection Coefficient, SWR and Return Loss

| Power Ratio ($P_R$ /$P_F$) | Reflection Coefficient (ρ) | Standing Wave Ratio (SWR) | Return Loss (RL) in dB |
|---|---|---|---|
| 0.0 | 0.00 | 1.0 | ∞ |
| 0.05 | 0.22 | 1.6 | 13.01 |
| 0.1 | 0.32 | 1.9 | 10.00 |
| 0.2 | 0.45 | 2.6 | 6.99 |
| 0.3 | 0.55 | 3.4 | 5.23 |
| 0.4 | 0.63 | 4.4 | 3.98 |
| 0.5 | 0.71 | 5.8 | 3.01 |
| 0.6 | 0.77 | 7.9 | 2.22 |
| 0.7 | 0.84 | 11.2 | 1.55 |
| 0.8 | 0.89 | 17.9 | 0.97 |
| 0.9 | 0.95 | 38 | 0.46 |
| 1.0 | 1.00 | ∞ | 0.00 |

## Calculating SWR

SWR, for a given antenna installation, can usually be measured much more easily than it can be calculated since instruments to measure it (SWR meters) are readily available or are easily built by the average ham. There are times, though, when calculating the SWR for a particular circuit is desired. For example, designing a calibration test load to test an antenna analyzer. An example follows.

The first step in computing SWR is determining the reflection coefficient (ρ) from one of several formulas:

$$\rho = E_R/E_F = I_R/I_F = \mathrm{sqrt}(P_R/P_F) = (Z_L - Z_0) / (Z_L + Z_0)$$

Where:
$\rho$ = reflection coefficient
$E_R$ = reflected voltage
$E_F$ = forward (incident) voltage
$I_R$ = reflected current
$I_F$ = forward (incident) current
$P_R$ = power in the reflected wave
$P_F$ = power in the forward (incident) wave
$Z_L$ = the complex load impedance whose real part is $R_L$ and whose reactive part is $X_L$
$Z_0$ = the complex line characteristic impedance whose real part is $R_0$ and whose reactive part is $X_0$.

And

SWR = $(1+ |\rho|) / (1-|\rho|)$

For low-loss transmission lines, or where (in the case of a calibration test load) there is no transmission line, the characteristic impedance $Z_0$ is almost completely resistive so that $Z_0$ is approximately $R_0$ and $X_0$ is approximately 0. The magnitude of $\rho$ then simplifies to:

$|\rho| = \text{sqrt}[((R_L – R_0)^2 + X_L^2) / ((R_L + R_0)^2 – X_L^2)]$

$R_0$ is the resistance the antenna analyzer expects for SWR=1
$R_L$ is the series resistance of the test load
$X_L$ is the series reactance of the test load

Lets use this information to design a calibration test load that might be used to check the accuracy of an antenna analyzer:

I’m going to assume that $R_0$ = 50 ohms, the reference impedance for most analyzers. The test load will be a series RC circuit and I’m going to arbitrarily make the resistive part $R_L$ = 50 ohms and the reactive part $X_L$ = –j50 ohms at 28 MHz.

$|\rho| = \text{sqrt}[((50 – 50)^2 + (-j50)^2) /((50 + 50)^2 – (-j50)^2)]$

$|\rho| = \text{sqrt}((0 + 2500) / ((100)^2 +2500)) = \text{sqrt}(2500/12500) = \text{sqrt}(0.2) = 0.4472$

SWR = (1 + 0.4472) / (1 - 0.4472) = 2.618

The value of the capacitor can be calculated from $Xc = 2\pi fC$. C = 115 pF.

I have 12 calibration loads like this that I built from surface mount components mounted on male BNC connectors. I got the idea from the November 2006 QST magazine. Any antenna analyzer should be evaluated using a complex (there’s that word again, Z = R ± jX) load such as this because that’s the real world. Simply connecting a 50-ohm resistor to the analyzer and seeing a SWR of 1 doesn't cut it. It's beyond the scope of this article, but before buying any antenna analyzer one should read some of the technical (as opposed to rah-rah non-technical) reviews available on the internet.

## Transmission Line Properties

Certain transmission line lengths have interesting properties:

1. An open-circuited quarter-wavelength transmission line looks like a short circuit at its free end.
2. A short-circuited quarter-wavelength transmission line looks like an open circuit at its free end.
3. A terminated half-wavelength transmission line repeats the terminating impedance at its free end.
4. A short-circuited transmission line shorter than one-quarter wavelength appears inductive at its free end.
5. An open-circuited transmission line shorter than one-quarter wavelength appears capacitive at its free end.

Lengths of transmission line that are exact multiples of a quarter-wavelength have the properties of resonant circuits. With an open-circuit termination, the input impedance of the line acts like a series resonant circuit. With a short-circuit termination, the line input simulates a parallel resonant circuit. (The ARRL Antenna Book, 21st Edition, page 24-14).

Always remember that the physical length of a transmission line is shorter than the calculated free-space electrical length because the velocity of propagation is slower in the transmission line and the time required for a signal of a given frequency to travel down a length of transmission line is longer than the time required for the same signal to travel the same distance in free space. This delay is expressed in terms of the speed of light, and is referred to as the velocity factor (VF) of the transmission line. (The ARRL Antenna Book, 21st Edition, page 24- 3).

For example, the calculated electrical length of a transmission line one-wavelength long at 10 MHz in free space (VF=1) is:
a) 299.7925/10 or approximately 30 meters or
b) 983.5712/10 or approximately 98.4 feet.
The physical length of the transmission line would be 25.5 meters or 83.6 feet for VF=0.85, typical for most RG-8 coax (a meter is 39.37 inches).

If the electrical length of the transmission line is critical, the VF of the exact piece of transmission being used should be determined experimentally (more on this later). Alternatively, the length can be 'trimmed' while watching the results on an antenna analyzer. The VF varies somewhat with frequency and even within production runs of the same cable.

## Antenna Effects

The chances of a transmission line being an exact impedance match to an antenna is somewhere between zero and none. A dipole antenna, for example, has a theoretical free-space impedance of 73 ohms at resonance. As the frequency applied to the dipole is varied away from resonance, however, a reactive component appears. When the frequency is greater than resonance, then the antenna tends to look like an inductive reactance, so the impedance is Z = R + jX. Similarily, when the frequency is less than the resonance frequency, the antenna looks like a capacitive reactance, so the impedance is Z = R – jX. Also, at distances closer to the earth's surface the resistive component may not be exactly 73 ohms, but may vary from about 30 to 130 ohms. (From the Practical Antenna Handbook, Fourth Edition, page 458).

## Transmission Line Effects

When installing an HF antenna, we usually just grab whatever coax is available and everything works fine. Taking the time to characterize that coax while both ends are accessible can pay dividends in the future, since you'll be able to tell much more about the antenna after everything in place. As I stated in Part I, what you see at the transmitter end of the transmission line probably isn't what's happening at the antenna, and you may be able to gain some performance by knowing the difference.

The impedance transforming properties of the transmission line can lead to erroneous conclusions about what's actually happening at the antenna. The data in Table 1 is from a 10-meter inverted V antenna fed with a 74-foot length of Columbia 1188 coax. The numbers in this example are taken from the ARRL Antenna Book, 21st Edition, page 27-31. The 'measured' values of R and X are as seen at the free (transmitter) end of the transmission line. The 'corrected' data shows the values present where the transmission line connects to the antenna, i.e., the impedance transformation effect of the transmission line has been removed.

Table 1 – Measured (uncorrected) and Corrected Data for 10-Meter Antenna Example

| Data Point | Freq (MHz) | Measured R | Measured X | Corrected R | Corrected X |
|---|---|---|---|---|---|
| 1 | 27.0 | 44 | 31.5 | 24 | -65 |
| 2 | 27.2 | 60 | 34.9 | 26 | -56 |
| 3 | 27.4 | 75 | 31.0 | 30 | -51 |
| 4 | 27.6 | 90 | 14.5 | 32 | -42 |
| 5 | 27.8 | 90 | -7.2 | 35 | -34 |
| 6 | 28.0 | 75 | -20.7 | 38 | -24 |
| 7 | 28.2 | 65 | -23.0 | 40 | -19 |
| 8 | 28.4 | 56 | -18.3 | 44 | -12 |
| 9 | 28.6 | 50 | -14.0 | 44 | -6 |
| 10 | 28.8 | 48 | -6.9 | 47 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 11 | 29.0 | 50 | 0 | 52 | 8 |
| 12 | 29.2 | 55 | 6.8 | 57 | 15 |
| 13 | 29.4 | 64 | 10.2 | 63 | 21 |
| 14 | 29.6 | 78 | 6.8 | 75 | 26 |
| 15 | 29.8 | 85 | 0 | 78 | 30 |
| 16 | 30.0 | 90 | -16.7 | 89 | 33 |

Looking at the measured data, it appears that the antenna has three resonant frequencies (where the reactance is zero): one between 27.6 MHz and 27.8MHz, one at 29.0 MHz and one at 29.8 MHz. That's an unlikely condition. The data is plotted in Fig. 1.

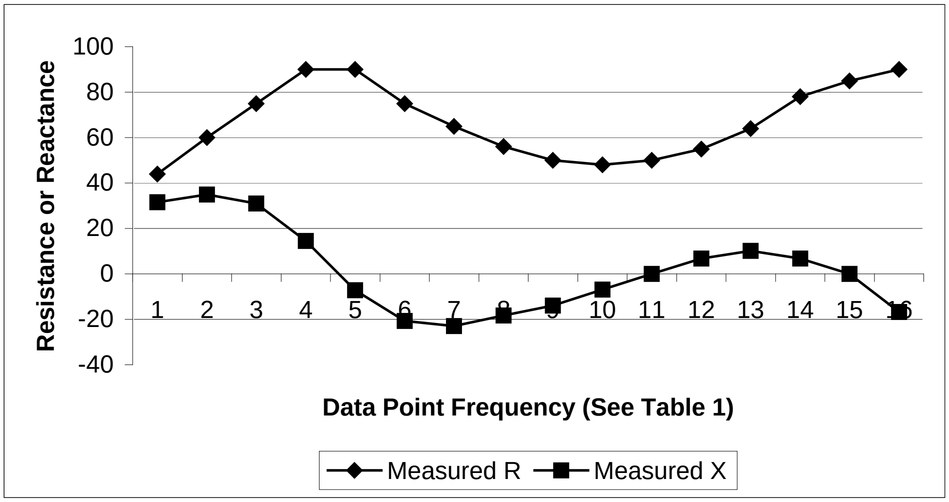

Fig.1 – Plot of uncorrected data from Table 1.

The corrected data, where the transforming effect of the transmission line has been removed, shows the "real" antenna with a single resonant frequency between 28.6 and 28.8 MHz (between data points 9 and 10 where the reactance X goes through zero). See Fig. 2. Obviously, in this case, the antenna parameters as seen at the free end of the transmission line are nothing like the parameters seen at the antenna.

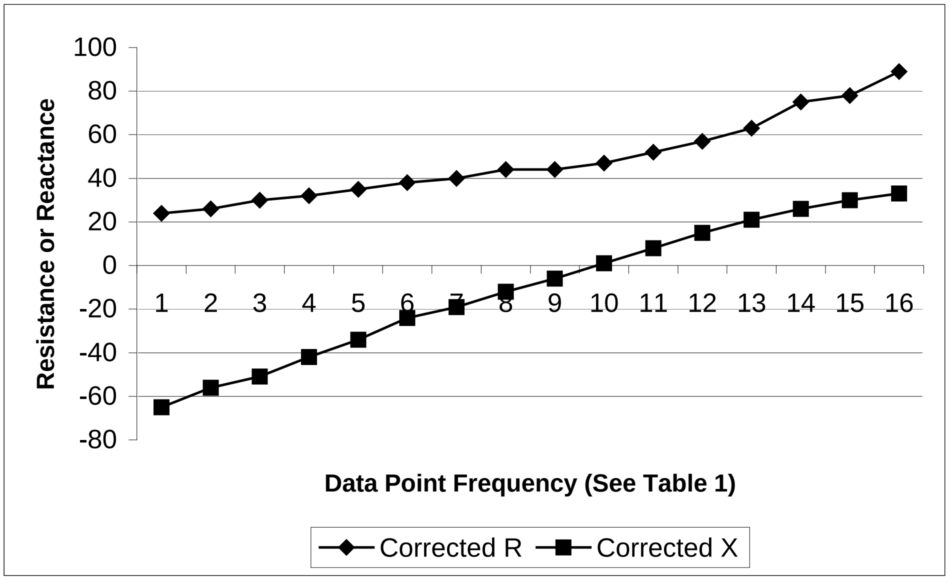

Fig. 2 – Plot of corrected data from Table 1.

More next month … stay tuned.